Solving the swing-up and balance task for the Acrobot and Pendubot with SAC

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Abstract— **We present a solution of the swing-up and balance task for the pendubot and acrobot for the participation in the AI Olympics competition at IJCAI 2023. Our solution is based on the Soft Actor Crtic (SAC) reinforcement learning (RL) algorithm for training a policy for the swing-up and entering the region of attraction of a linear quadratic regulator (LQR) controller for stabilizing the double pendulum at the top position. Our controller achieves competitive scores in performance and robustness for both, pendubot and acrobot, problem scenarios.**

I. Introduction

Robotic control is a complex problem and is approached in many different ways, such as classical motion planning, optimal control methods and reinforcement learning methods. The AI Olympics competition at IJCAI 2023 is an attempt to compare control methods in a simple but challenging setting. The competition involves the application of the control methods not only in a simulation setting but also on a real combined pendubot and acrobot hardware system.

Extensive research has been conducted on complex robotic motion control, with learning-based methods gaining widespread adoption in recent times. A notable example is the classical control module within the Gymnasium environment [1], a well-known reinforcement learning framework developed by OpenAI. This module serves as an ideal platform for exploring and evaluating new motion control algorithms. Another noteworthy achievement in the field is the DeepMimic project [2]. Through imitation learning within a simulated environment, this project has enabled robots to acquire intricate human movements, including backflips and martial arts maneuvers. Additionally, a research team from ETH Zurich has made significant progress in addressing the sim-to-real interface challenge [3]. Their methodology involves training a control model for a mini-cheetah-like robot within a simulation environment. By utilizing a neural network and leveraging data collected from the real robot, they approximated the dynamics model of the physical robot. This approach has facilitated accurate implementation of the control policy derived from the virtual environment onto the real robot. These exemplary works demonstrate the promising applications of learning-based approaches in various aspects of robot control.

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The AI Olympics challenge focuses on two underactuated variations of the double pendulum model, namely the pendubot and acrobot. The objective revolves around accomplishing two primary tasks: swinging the double pendulum from its lowest position to its highest point, and maintaining stability at the highest point. To achieve the swing-up task, we employ the classical model-free reinforcement learning algorithm known as Soft Actor-Critic (SAC) [4] to train a policy which is able to reach the region of attraction (RoA) of a continuous time linear quadratic regulator (LQR) controller [5], [10]. As soon as the system enters the RoA, we transition to the LQR controller to stabilize the entire system.

This paper is structured as follows: In the next section, we briefly summarize the SAC algorithm. In Section III, we explain the reward function that we used for the training, how we conducted the training and how we composed the controller. We present the results in Section IV and conclude in Section V.

II. Soft-Actor-Crtic

Soft Actor Critic (SAC) [4] is a popular algorithm used in the field of reinforcement learning. It is originally designed for continuous action spaces, where the agent has an infinite choice of actions to take. In our problem scenario, the actuators of the double pendulum can be set to any value within the torque limit range. The position and velocity measurements obtained from the motors are also represented as continuous real numbers. Therefore, we choose the SAC algorithm to train the agent.

SAC optimizes a policy by maximizing the expected cumulative reward obtained by the agent over time. This is achieved through an actor and critic structure [6].

The actor is responsible for selecting actions based on the current policy in response to the observed state of the environment. It is typically represented by a shallow neural network that approximates the mapping between the input state and the output probability distribution over actions. SAC incorporates a stochastic policy in its actor part, which encourages exploration and helps the agent improve policies.

The critic, on the other hand, evaluates the value of state-action pairs. It estimates the expected cumulative reward that the agent can obtain by following a certain policy. Typically, the critic is also represented by a neural network that takes state-action pairs as inputs and outputs estimated value.

In addition to the actor and critic, a central feature of SAC is entropy regularization[7]. The policy is trained to maximize a trade-off between expected return and entropy, which is a measure of randomness in the action selection. If x is a random variable with a probability density function P, the entropy H of x is defined as:

$$H(P) = \mathop{\mathbb{E}}_{x \sim P}[-\log P(x)]$$

By maximizing entropy, SAC encourages exploration and accelerates learning. It also prevents the policy from prematurely converging to a suboptimal solution. The trade-off between maximizing reward and maximizing entropy is controlled through a parameter, α. This parameter serves to balance the importance of exploration and exploitation within the optimization problem. The optimal policy π^* can be defined as follows:

$$\pi^* = arg\max_{\pi} \mathop{\mathbb{E}}_{\tau \sim \pi}\left[\sum_{t=0}^{\infty} \gamma^t \Big(R(s_t, a_t, s_{t+1}) + \alpha H(\pi(\cdot \mid s_t))\Big)\right]$$

During training, SAC learns a policy π_θ and two Q-functions Q_{ϕ_1}, Q_{ϕ_2} concurrently. The loss functions for the two Q-networks are $(i \in 1, 2)$:

$$L(\phi_i, D) = \mathop{\mathbb{E}}_{(s,a,r,s',d) \sim D}\left[\left(Q_{\phi_i}(s, a) - y(r, s', d)\right)^2\right],$$

where the temporal difference target y is given by:

$$\begin{aligned} y(r, s', d) = r + \gamma(1 - d) \times \\ \left(\min_{j=1,2} Q_{\phi_{targ,j}}(s', \tilde{a}') - \alpha \log \pi_\theta(\tilde{a}' \mid s')\right), \\ \tilde{a}' \sim \pi_\theta(\cdot \mid s') \end{aligned}$$

In each state, the policy π_θ should act to maximize the expected future return Q while also considering the expected future entropy H. In other words, it should maximize $V^\pi(s)$:

$$\begin{aligned} V^\pi(s) &= \mathop{\mathbb{E}}_{a \sim \pi}[Q^\pi(s, a)] + \alpha H(\pi(\cdot \mid s)) \\ &= \mathop{\mathbb{E}}_{a \sim \pi}[Q^\pi(s, a)] - \alpha \log \pi(a \mid s) \end{aligned}$$

By employing an effective gradient-based optimization technique, the parameters of both the actor and critic neural networks undergo updates, subsequently leading to the adaptation of the policies themselves.

In conclusion, SAC's combination of stochastic policies, exploration through entropy regularization, value estimation, and gradient-based optimization make it a well-suited algorithm for addressing the challenges posed by continuous state and action spaces.

III. Controller

Pendubot and acrobot represent two variations of the double pendulum, distinguished by which joint is actuated. In the case where the shoulder joint is actuated, it is referred to as pendubot, while the term acrobot is used when the elbow joint is actuated. To conduct our study, we used a custom simulation environment building up on the simulation developed by the competition organizers [8]. The environment is constructed based on the dynamics provided in the double pendulum repository for the competition, and it incorporates standard OpenAI Gym environment features. We used the vanilla SAC algorithm implemented in Stable Baseline3 [9].

The simulation environment encompasses four controlled variables: the angular position (p_1) and angular velocity (v_1) of the shoulder joint, as well as the angular position (p_2) and angular velocity (v_2) of the elbow joint. The whole state is $x = [p_q, p_2, v_1, v_2]^T$. The controller generates output u which corresponds to the torque applied on the shoulder τ_1 for the pendubot and to the torque applied on the elbow τ_2 for the acrobot. In order to facilitate effective training of the agent, we found it beneficial to use normalized state and action representations. This mapping between real-world physical quantities (x, u) and the agent's state and action space (s, a) are

$$a = \tau_{max} u \tag{1}$$

$$s_i = \frac{(p_i \mod 2\pi) - \pi}{\pi} \quad , i \in \{1, 2\} \tag{2}$$

$$s_i = \frac{\min(\max(x_i, -v_{max}), v_{max})}{v_{max}} \quad , i \in \{3, 4\} \tag{3}$$

We used $\tau_{max} = 5.0\,\mathrm{Nm}$ and $v_{max} = 20.0\,\mathrm{rad/s}$.

In theory, the reward function is designed to guide the agent's behavior towards achieving stability around the system's goal point. However, practical implementation has revealed challenges such as potential entrapment in local minima or difficulty in maintaining stability at the highest point for extended periods. To address these issues, we employ two main approaches. For the stabilization problem, we introduce the concept of a combined controller. During the swing-up process, the RL-trained agent assumes control, leveraging its learned policies. However, as the system approaches proximity to the maximum point, a smooth transition occurs, allowing a continuous time LQR controller to provide the final stabilization necessary for maintaining stability at the highest point.

To steer the agent away from hazardous local minima, we devised a three-stage reward function. The full equation for this reward function is

$$\begin{aligned} r(x, u) = &-(x - x_g)^T Q_{train}(x - x_g) - u^T R_{train} u \\ &+ \begin{cases} r_{line} & \text{if } h(p_1, p_2) \geq h_{line} \\ 0 & \text{else} \end{cases}, \\ &+ \begin{cases} r_{LQR} & \text{if } (x - x_g)^T S_{LQR}(x - x_g) \geq \rho \\ 0 & \text{else} \end{cases}, \\ &- \begin{cases} r_{vel} & \text{if } |v_1| \geq v_{thresh} \\ 0 & \text{else} \end{cases}, \\ &- \begin{cases} r_{vel} & \text{if } |v_2| \geq v_{thresh} \\ 0 & \text{else} \end{cases}, \end{aligned} \tag{4}$$

In the initial stage, a quadratic reward function is employed to encourage smooth swinging of the entire system

Robot	Quadratic Reward	Constant Reward	LQR
Pendubot	Q_1 = 8.0 Q_2 = 5.0 Q_3 = 0.1 Q_4 = 0.1 R = 1e-4	$r_{line} = 500$ $r_{vel} = 0.0$ $r_{LQR} = 1e4$	Q_1 = 1.92 Q_2 = 1.92 Q_3 = 0.3 Q_4 = 0.3 R = 0.82
Acrobot	Q_1 = 10.0 Q_2 = 10.0 Q_3 = 0.2 Q_4 = 0.2 R = 1e-4	$r_{line} = 500$ $r_{vel} = 1e4$ $r_{LQR} = 1e4$	Q_1 = 0.97 Q_2 = 0.93 Q_3 = 0.39 Q_4 = 0.26 R = 0.11

TABLE I

HYPER PARAMETERS USED FOR THE SAC TRAINING AND THE LQR CONTROLLER.

Criteria	Pendubot	Acrobot
Swingup Success	success	success
Swingup time [s]	0.65	2.06
Energy [J]	9.4	29.24
Max. Torque [Nm]	5.0	5.0
Integrated Torque [Nm]	2.21	4.57
Torque Cost [N²m²]	8.58	12.32
Torque Smoothness [Nm]	0.172	0.954
Velocity Cost [m²/s²]	44.98	193.78
RealAI Score	0.801	0.722

TABLE II

PERFORMANCE SCORES OF OUR CONTROLLER FOR PENDUBOT AND ACROBOT.

within a relatively small number of training sessions. The matrix $Q_{train} = diag(Q_1, Q_2, Q_3, Q_4)$ is a diagonal matrix, while R_{train} is a scalar. This is due to the nature of underactuated control in the double pendulum system, where only a single control input is available.

As the end effector reaches a threshold line $h_{line} = 0.8(l_1 + l_2)$, we introduce a second level of reward r_{line}. The end effector height is given by

$$h(p_1, p_2) = -l_1 \cos(p_1) - l_2 \cos(p_1 + p_2). \tag{5}$$

with the link lengths l_1 and l_2. This reward provides the agent with a fixed value but is carefully designed to prevent the system from spinning rapidly in either clockwise or counterclockwise directions. To discourage the agent from exploiting rewards by spinning at excessive speeds, a significant penalty $-r_{vel}$ is implemented for any speed exceeding $v_{thresh} = 8\,\text{rad/s}$ in absolute value. This penalty effectively compels the agent to approach the maximum point while adhering to the predefined speed interval. The speed penalty was only needed for the acrobot.

The third level of reward r_{LQR} aims to provide a substantial reward to the agent when it remains within the Region of Attraction (ROA) of the LQR controller. By this we want to achieve that the policy learns to enter the LQR controller RoA so that there can be a smooth transition between both controllers. For details on the LQR controller and its region of attraction, we refer to these lecture notes [10]. The parameters, we used in the cost matrices of the LQR controller are listed in Table I. We computed the RoA similar to [11] but with a sums of squares method [12]. Once the RoA is computed, it can be checked whether a state x belongs to the estimated RoA of the LQR controller by calculating the cost-to-go of the LQR controller with the matrix S_{LQR} and comparing it with the scalar ρ.

The parameters we used for the reward function and the LQR controller are listed in Table I.

During the training phase of the SAC controller for both the acrobot and pendubot, we placed our focus on tuning several crucial hyperparameters. These hyperparameters included the learning rate, control frequency, episode length, and learning time steps. We carefully set the learning rate to 0.01 to facilitate effective learning and adaptation. Additionally, the control frequency of the simulation was set to 100Hz, ensuring frequent updates and responsiveness in the control process. For each training episode, we chose an episode length of 1000 (Acrobot) and 500 (Pendubot) corresponding to 10 s and 5 s long episodes to provide sufficient exploration and learning opportunities. To maximize the training potential, we conducted a total of $2e7$ learning time steps, allowing the agent to gather extensive experience and refine its performance.

During the execution, at every control step, it is checked whether the current state belongs to the estimated RoA of the LQR controller. If it does the LQR controller command is used else the SAC policy command is used.

We encountered heavy oscillation in the simulation when the LQR controller took over from the SAC controller. This oscillation was primarily caused by the low control frequency. To address this issue and maintain stability, we adjusted the control frequency to 500Hz, enabling smoother control and reducing the oscillation experienced by the LQR controller.

IV. RESULTS

Our carefully designed pendubot controller delivered commendable outcomes in both the swing-up and stabilization tasks. With a swing-up time metric of 0.65 seconds, our pendubot controller achieved a performance level that matches the top result on the leaderboard by the time-varying LQR controller. The swing-up trajectory is visualized in Fig. 1. Most other performance metrics (Table II), such as Energy, Integrated Torque, and Velocity Cost, demonstrated impressive performance, with the exception of Torque smoothness, where the controller showed some room for improvement.

In terms of robustness, the controller surpassed our expectations with a score of 0.916 (Fig. 3(a)). It demonstrated near-perfect resilience to measurement noise, torque noise, and torque response. Additionally, the controller outperformed the majority of other controllers when faced with challenges such as time delays and model inaccuracies. The combined score of performance and robustness for the pendubot is 0.86.

The performance evaluation of our trained model on the acrobot task reveals its success in both swing-up and stabilization tasks (trajectory in Fig. 2, performance scores in Table II). Particularly noteworthy is its swing-up time of only 2.06 seconds, giving it a clear advantage in terms of

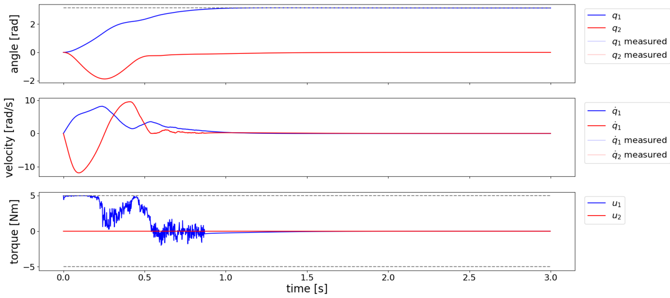


Fig. 1. Swing-up trajectory of the pendubot.

speed. Although our model delivers fair results in terms of maximum torque, integrated torque, and torque cost, we have made some compromises in terms of torque smoothness and energy efficiency.

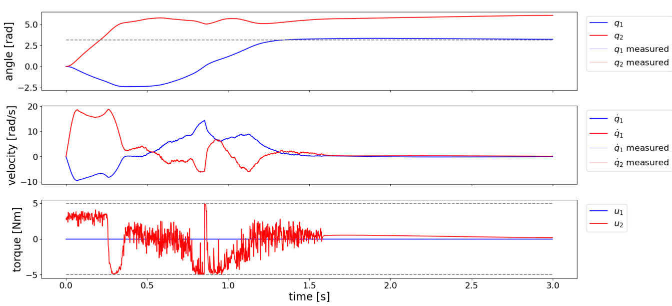
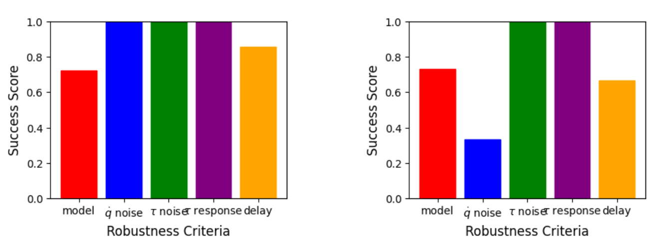


Fig. 2. Swing-up trajectory of the Acrobot.

When it comes to robustness, our trained model proves resilient against torque noise and demonstrates commendable performance in torque response (Fig. 3(b)). It also holds up reasonably well against model inaccuracies and time delays. However, it exhibits high sensitivity when encountering measurement noise. The overall robustness score is 0.747 and the average with the performance score is 0.73.



(a) Pendubot (b) Acrobot

Fig. 3. Robustness results of our controller for pendubot and acrobot.

V. Conclusion

The combination of an RL-based controller and a continuous time LQR controller proved to be successful in controlling both the acrobot and pendubot systems for swing-up and stabilization tasks. Notably, both variations exhibited commendable swing-up time metrics and demonstrated robustness. However, there were two main challenges encountered during the experiment.

The first challenge involved establishing a smooth switching condition between the RL-based controller and the LQR controller. This necessitated optimizing the region of attraction specific to the LQR controller, ensuring a seamless transition between the two control approaches. The second challenge revolved around training a stable agent using a model-free approach, specifically SAC, to ensure that the entire system approached the target point at a sufficiently low speed to enter the region of attraction where the LQR controller could take over.

To overcome these challenges, we developed a three-stage reward function to interface with the agent. However, the process of fine-tuning the hyperparameters for this reward function proved to be quite time-consuming.

Throughout our experiments, we have come very close to relying solely on the RL-based controller for both swing-up and stabilization tasks. In the future, our plans involve enhancing the existing reward function, which will increase the likelihood of training an agent capable of achieving self-stabilization around a desired position. Additionally, we will focus our efforts on further approximating the training environment to closely resemble real-world physics. This will facilitate a smoother transition of the controller from the simulated environment to the actual physical system.

References

[1] G. Brockman, V. Cheung, L. Pettersson, J. Schneider, J. Schulman, J. Tang, and W. Zaremba, "Openai gym," *arXiv preprint arXiv:1606.01540*, 2016.

[2] X. B. Peng, P. Abbeel, S. Levine, and M. van de Panne, "Deepmimic: Example-guided deep reinforcement learning of physics-based character skills," *ACM Trans. Graph.*, vol. 37, no. 4, pp. 143:1–143:14, Jul. 2018. [Online]. Available: http://doi.acm.org/10.1145/3197517.3201311

[3] J. Hwangbo, J. Lee, A. Dosovitskiy, D. Bellicoso, V. Tsounis, V. Koltun, and M. Hutter, "Learning agile and dynamic motor skills for legged robots," *Science Robotics*, vol. 4, no. 26, p. eaau5872, 2019.

[4] T. Haarnoja, A. Zhou, P. Abbeel, and S. Levine, "Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor," in *International conference on machine learning*. PMLR, 2018, pp. 1861–1870.

[5] N. Lehtomaki, N. Sandell, and M. Athans, "Robustness results in linear-quadratic gaussian based multivariable control designs," *IEEE Transactions on Automatic Control*, vol. 26, no. 1, pp. 75–93, 1981.

[6] V. Konda and J. Tsitsiklis, "Actor-critic algorithms," *Advances in neural information processing systems*, vol. 12, 1999.

[7] J. Achiam, "Spinning Up in Deep Reinforcement Learning," 2018.

[8] F. Wiebe, S. Kumar, L. Shala, S. Vyas, M. Javadi, and F. Kirchner, "An open source dual purpose acrobot and pendubot platform for benchmarking control algorithms for underactuated robotics," *IEEE Robotics and Automation Magazine*, 2023, under review.

[9] A. Raffin, A. Hill, A. Gleave, A. Kanervisto, M. Ernestus, and N. Dormann, "Stable-baselines3: Reliable reinforcement learning implementations," *Journal of Machine Learning Research*, vol. 22, no. 268, pp. 1–8, 2021. [Online]. Available: http://jmlr.org/papers/v22/20-1364.html

[10] R. Tedrake, *Underactuated Robotics*, 2022. [Online]. Available: http://underactuated.mit.edu

[11] L. Maywald, F. Wiebe, S. Kumar, M. Javadi, and F. Kirchner, "Co-optimization of acrobot design and controller for increased certifiable stability," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS-2022)*, 10 2022.

[12] R. Tedrake, I. R. Manchester, M. Tobenkin, and J. W. Roberts, "LQR-trees: Feedback Motion Planning via Sums-of-Squares Verification," *The International Journal of Robotics Research*, vol. 29, no. 8, pp. 1038–1052, Jul. 2010.